The Capacitances and Surface-Charge Distributions of a Shielded Balanced Pair

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Abstract—The capacitance matrix of a straight pair of uniform wires symmetrically placed in a shield is determined theoretically.

Exact expressions for the elements of the capacitance matrix are determined as particular elements of the inverse of an infinite matrix which relates the Fourier coefficients of the surface-charge densities on the inner conductors and the shield to the applied voltage excitations on the cable conductors. If the wire diameter is small relative to the wire separation, and if the wire separation is small relative to the shield diameter, then accurate numerical approximations for the elements of the capacitance matrix are obtained to any degree of accuracy by suitably truncating the infinite matrix.

Once the elements of the capacitance matrix are determined, then the distributions of the surface-charge densities on the peripheries of the inner conductors and the shield are determined for any arbitrary excitation of the cable structure. In particular, the various capacitances associated with the cable structure, e.g., the direct, ground, and mutual capacitances, are determined from a comparison of the surface-charge densities resulting from a "balanced" excitation and a "longitudinal" excitation.

The Fourier coefficients of the surface-charge densities are required to determine the propagation parameters and the associated propagation modes of the cable structure. The surface-charge distributions are evaluated numerically for a typical standard production cable using 22-gauge wires.

The results of this paper will be extended by a perturbational method to include twisted wires in a shield; also, certain types of asymmetries in the cable geometry will be considered. Hence, the propagation constants and the associated propagation modes of unbalanced and/or twisted shielded pair cables can also be determined.

I. INTRODUCTION

THE capacitance matrix (per unit length in the axial direction) is determined for a straight pair of wires in a shield. The capacitance matrix relates the Fourier coefficients of the surface-charge densities on the inner conductors and the shield to the voltage excitations applied to the cable conductors.

Once the elements of the capacitance matrix are determined, then the distributions of the surface-charge densities on the peripheries of the inner conductors and the shield are determined for any arbitrary excitation of the cable structure. These Fourier coefficients can be used to determine the propagation parameters and the associated propagation modes of the cable structure. The method for doing so, together with applications, will be presented in a forthcoming paper.

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In previous papers on this subject [1]–[3], only the various capacitances associated with the cable structure, e.g., the direct, ground, and mutual capacitances, were determined. These capacitances were determined indirectly from a consideration of only the case of "balanced" or "longitudinal" excitation, without directly calculating the Fourier coefficients of the various surface-charge densities involved.

For balanced or longitudinal excitation, the cable structure and the resulting electrostatic potential distribution within the shield are symmetric about a line passing through the axis of the shield perpendicular to the center line of the wires; and, therefore, the surface-charge densities on the inner conductors are images of each other. The general case of arbitrary excitation can be expressed as the appropriate superposition of a balanced excitation and a longitudinal excitation; however, the use of the aforementioned symmetry argument is not invoked in this study since the results of this study are to be applied later to the case of a lossy unbalanced twisted shielded-pair cable for which the "propagating" modes are no longer the balanced and longitudinal modes.

In this paper, the previous methods for determining the elements of the capacitance matrix are extended so that the Fourier coefficients of the surface-charge densities on the peripheries of the inner conductors and the shield are also determined. In the process of obtaining the Fourier coefficients of the various surface-charge densities, the various capacitances associated with the cable structure are also determined. In addition, the voltage excitations on the inner conductors and the shield are assumed to be completely arbitrary; however, the cable structure itself is constrained to be symmetric about the axis of the shield, i.e., the wires are symmetrically located about the axis of the shield and have the same radii.

The surface-charge densities on each wire and the shield are functions of only the azimuthal angles describing the circumferences of each wire and the inner circumference of the shield, and are expanded in Fourier series in these azimuthal angles. The resulting electrostatic potential distribution within the cable structure is also represented in a Fourier series in these azimuthal angles.

The Fourier coefficients of the electrostatic potential distribution are related by Laplace's equation to the Fourier coefficients of the surface-charge densities on the inner conductors and the shield. The solution of Laplace's equation, subject to the boundary conditions impressed by the applied voltage excitations on the cable conductors, gives rise to an infinite matrix which relates the Fourier coefficients of the surface-charge densities to the applied voltage excitations on the inner conductors and the shield.

Exact expressions for the elements of the capacitance matrix are then determined as particular elements of the inverse of the infinite matrix. If the wire radius is small relative to the wire spacing and if the wire spacing is small relative to the shield radius, then accurate numerical approximations for the elements of the capacitance matrix are obtained to any degree of accuracy by suitably truncating the infinite matrix.

This solution for the case of an untwisted cable will be used later as the zeroth-order solution in a perturbational analysis to determine the corresponding results for the twisted cable. Also, the model will be extended to include certain types of asymmetries in the cable geometry. Hence, the results of this paper can also be applied to numerically evaluate the propagation parameters and the associated propagation modes of unbalanced and/or twisted shielded-pair cables.

II. CABLE GEOMETRY

The geometry of the cable is shown in Fig. 1. The cable consists of two straight spatially separated cylindrical inner conductors embedded in a simple insulator (i.e., a linear, homogeneous, isotropic, and time-invariant medium), which are enclosed by a conducting annular shield. It is assumed that the centers of the wires are spaced equidistantly at a distance s on the same line from the center of the shield. It is assumed that the wires are of the same circular cross section with radius δ and are composed of the same conducting materials. The dielectric in which the wires are embedded completely surrounds each wire and extends uniformly out to the inner radius Δ of the annular shield. The dielectric is determined by its constitutive parameters ϵ and μ. It is assumed that the conductivities of each wire and the shield are infinite and that conductivity of the dielectric is zero. These conductivity assumptions, when used to determine the surface-charge distributions on the various conductors, can be shown to be reasonable and do not significantly affect the accuracy of the solution in the frequency range of interest in this study.

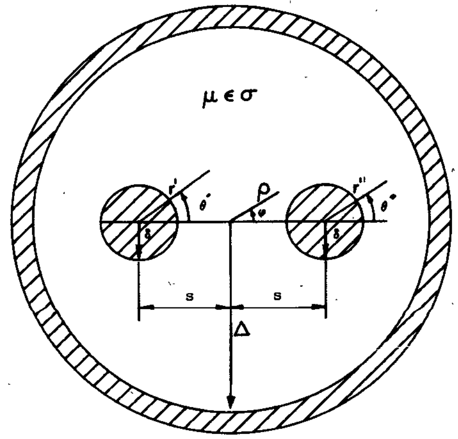

Fig. 1. Cross section of the shielded-pair cable.

Dimensional restrictions are imposed on the parameters of the cable to keep the three conductors of the structure from touching, i.e., let

$$s > \delta$$

$$\Delta > s + \delta.$$

III. THEORY

In a cylindrical coordinate system (ρ,φ,z), let a surface-charge density σ be distributed over the cylinder $\rho = \rho_0$, $-\infty < z < +\infty$. Assume that the surface-charge density σ is not a function of the axial variable z, so that it may be represented by its Fourier series expansion in the azimuthal angle φ, i.e., let

$$\sigma \equiv \frac{\lambda_0}{2\pi\rho_0}\left[\xi_0 + \sum_{\iota=1}^{\infty} (\zeta_\iota \cos \iota\varphi + \eta_\iota \sin \iota\varphi)\right]$$

where ζ_ι, η_ι, and ξ_0 are the Fourier coefficients of the expansion, and λ_0 is a normalization constant.

Due to the axial symmetry, the potential Φ satisfies the transverse Poisson equation

$$\nabla_t^2\Phi = -\frac{1}{\epsilon}\sigma\delta(\rho - \rho_0)$$

where the transverse Laplacian ∇_t^2 in a cylindrical coordinate system is

$$\nabla_t^2 \equiv \frac{1}{\rho}\frac{\partial}{\partial\rho}\rho\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}.$$

Then,

$$\Phi = \Phi(\rho,\varphi \mid \sigma)$$

where [4]

$$\Phi(\rho,\varphi \mid \sigma) \equiv -\frac{1}{2\pi\epsilon}\lambda_0\left[\xi_0 \ln \rho_> - \tfrac{1}{2}\sum_{\iota=1}^{\infty}\left(\frac{\rho_<}{\rho_>}\right)^{\iota} \cdot \frac{\zeta_\iota \cos \iota\varphi + \eta_\iota \sin \iota\varphi}{\iota}\right]$$

and $\rho_<$ and $\rho_>$ are, respectively, the lesser and the greater values of ρ_0 and ρ.

Since each conductor is described by its own local coordinate system, each coordinate system with a different polar origin (see Fig. 1), several coordinate transformations are required to provide a complete solution which satisfies all the boundary conditions at the surfaces of each conductor.

Consider, then, a translated cylindrical coordinate system (r,θ,z), with polar origin at $\rho = c$, $\varphi = 0$, and initial line $\varphi = 0$, where $c > \rho_0$. The potential Φ in the transverse plane (r,θ) due to the surface-charge density σ transforms to

$$\Phi = \begin{cases} \bar{\psi}(r,\theta;c \mid \sigma), & r < c - \rho_0 \\ \tilde{\psi}(r,\theta;c \mid \sigma), & r > c + \rho_0 \end{cases}$$

where [4]

$$\bar{\psi}(r,\theta;c \mid \sigma) \equiv -\frac{1}{2\pi\epsilon}\lambda_0\Bigg[\xi_0 \ln \mid c \mid - \xi_0 \sum_{\iota=1}^{\infty} (-)^{\iota}\left(\frac{r}{c}\right)^{\iota}\frac{\cos \iota\theta}{\iota} - \tfrac{1}{2}\sum_{\iota=1}^{\infty}\sum_{\iota'=0}^{\infty} (-)^{\iota'}\, {}^{\iota}\beta_{\iota'}\left(\frac{\rho_0}{c}\right)^{\iota}\left(\frac{r}{c}\right)^{\iota'} \cdot \frac{\zeta_{\iota}\cos \iota'\theta - \eta_{\iota}\sin \iota'\theta}{\iota}\Bigg]$$

and [4]

$$\tilde{\psi}(r,\theta;c \mid \sigma) \equiv -\frac{1}{2\pi\epsilon}\lambda_0\Bigg[\xi_0 \ln r - \xi_0 \sum_{\iota=1}^{\infty} (-)^{\iota}\left(\frac{c}{r}\right)^{\iota}\frac{\cos \iota\theta}{\iota} - \tfrac{1}{2}\sum_{\iota=1}^{\infty}\sum_{\iota'=0}^{\infty} (-)^{\iota'}\, {}^{\iota}\beta_{\iota'}\left(\frac{\rho_0}{r}\right)^{\iota}\left(\frac{c}{r}\right)^{\iota'} \cdot \frac{\zeta_{\iota}\cos (\iota + \iota')\theta + \eta_{\iota}\sin (\iota + \iota')\theta}{\iota}\Bigg]$$

where the binomial coefficient β is defined by

$${}^{\iota}\beta_{\iota'} \equiv \frac{(\iota - 1 + \iota')!}{(\iota - 1)!\,\iota'!}.$$

Similarly, consider a translated cylindrical coordinate system (r,θ,z), with polar origin at $\rho = c$, $\varphi = 0$, and initial line $\varphi = 0$, where $c < \rho_0$. The potential Φ in the transverse plane (r,θ) due to the surface-charge density σ transforms to

$$\Phi = \psi(r,\theta;c \mid \sigma) \qquad \begin{cases} r < c \\ r < \rho_0 - c \end{cases}$$

where [4]

$$\psi(r,\theta;c \mid \sigma) \equiv -\frac{1}{2\pi\epsilon}\lambda_0\Bigg[\xi_0 \ln \rho_0 - \tfrac{1}{2}\sum_{\iota=1}^{\infty}\sum_{\iota'=0}^{\iota} {}^{\iota}\gamma_{\iota'}\left(\frac{c}{\rho_0}\right)^{\iota}\left(\frac{r}{c}\right)^{\iota'} \cdot \frac{\zeta_{\iota}\cos \iota'\theta + \eta_{\iota}\sin \iota'\theta}{\iota}\Bigg]$$

where the combinatorial coefficient γ is defined by

$${}^{\iota}\gamma_{\iota'} \equiv \frac{\iota!/\iota'!}{(\iota - \iota')!}.$$

IV. APPLICATION

In terms of their local cylindrical coordinate systems, let the surface-charge density on each wire (referred to as conductor 1 or 2) denoted by the superscript $'$ or $''$, and the surface-charge density on the shield (referred to as conductor 3) denoted without a superscript, be expanded as

$$\sigma^{\left\{\substack{\prime\\ \prime\prime}\right.} = \frac{\lambda_0}{2\pi\delta}\left[c_0^{\left\{\substack{\prime\\ \prime\prime}\right.} + \sum_{\iota=1}^{\infty}\left(c_{\iota}^{\left\{\substack{\prime\\ \prime\prime}\right.}\cos \iota\theta^{\left\{\substack{\prime\\ \prime\prime}\right.} + k_{\iota}^{\left\{\substack{\prime\\ \prime\prime}\right.}\sin \iota\theta^{\left\{\substack{\prime\\ \prime\prime}\right.}\right)\right]$$

$$\sigma = \frac{\lambda_0}{2\pi\Delta}\left[s_0 + \sum_{\iota=1}^{\infty}(s_{\iota}\cos \iota\varphi + t_{\iota}\sin \iota\varphi)\right]$$

where $c_0^{\prime,\prime\prime}, c_{\iota}^{\prime,\prime\prime}, k_{\iota}^{\prime,\prime\prime}$ and $s_0, s_{\iota}, t_{\iota}$ are the Fourier coefficients of the expansions, and λ_0 is a normalization constant.

It is assumed that the inner conductors are excited with the voltages V_1 and V_2, and that the shield is excited with the voltage V_3, such that

$$V_{\left\{\substack{1\\ 2}\right.} = V_0 u_0^{\left\{\substack{\prime\\ \prime\prime}\right.}$$

$$V_3 = V_0 u_0$$

where $u_0^{\prime,\prime\prime}$ and u_0 are the Fourier coefficients of the expansion, and V_0 is a normalization constant.

Referring to Fig. 1, let Φ_{jk} denote the potential on the conductor (j) due to the surface-charge density on the conductor (k).

Therefore, on wire 1,

$$V_1 = \Phi_{11} + \Phi_{12} + \Phi_{13}$$

where

$$\Phi_{11} = \Phi(\delta,\theta' \mid \sigma')$$
$$\Phi_{12} = \bar{\Psi}(\delta,\theta'; -2s \mid \sigma'')$$
$$\Phi_{13} = \Psi(\delta,\theta'; -s \mid \sigma).$$

Similarly, on wire 2,

$$V_2 = \Phi_{21} + \Phi_{22} + \Phi_{23}$$

where

$$\Phi_{21} = \bar{\Psi}(\delta,\theta''; 2s \mid \sigma')$$
$$\Phi_{22} = \Phi(\delta,\theta'' \mid \sigma'')$$
$$\Phi_{23} = \Psi(\delta,\theta''; s \mid \sigma).$$

Also, on the shield

$$V_3 = \Phi_{31} + \Phi_{32} + \Phi_{33}$$

where

$$\Phi_{31} = \tilde{\Psi}(\Delta,\varphi; s \mid \sigma')$$
$$\Phi_{32} = \tilde{\Psi}(\Delta,\varphi; -s \mid \sigma'')$$
$$\Phi_{33} = \Phi(\Delta,\varphi \mid \sigma).$$

Superposing the various potential terms and equating them to the values of the voltage excitations for each value of the index ι yields an infinite set of equations in which the Fourier coefficients of the various surface-charge densities are the unknowns [4]. Notice that, due to the symmetry, the sine and cosine terms decouple, and all of the sine coefficients are zero. The final solution for the infinite sets of Fourier cosine coefficients of the various surface-charge densities is conveniently represented in terms of partitioned infinite matrices, i.e., let

$$-\frac{1}{2\pi\epsilon}\lambda_0 MX = V_0 E$$

or

$$X = -2\pi\epsilon\frac{V_0}{\lambda_0}M^{-1}E$$

where

$$M = \begin{bmatrix} O^{00} & R^{01} & R^{02} & R^{03} & \cdots \\ C^{10} & D^{11} & U^{12} & U^{13} & \\ C^{20} & L^{21} & D^{22} & U^{23} & \\ C^{30} & L^{31} & L^{32} & D^{33} & \\ \vdots & & & & \ddots \end{bmatrix}$$

and

$$O^{00} = \begin{bmatrix} \ln\delta & \ln|-2s| & \ln\Delta \\ \ln|2s| & \ln\delta & \ln\Delta \\ \ln\Delta & \ln\Delta & \ln\Delta \end{bmatrix}$$

and

$$C^{\iota 0} = \begin{bmatrix} 0 & \left(\frac{\delta}{2s}\right)^{\iota}\frac{1}{\iota} & 0 \\ (-)^{\iota}\left(\frac{\delta}{2s}\right)^{\iota}\frac{1}{\iota} & 0 & 0 \\ (-)^{\iota}\left(\frac{s}{\Delta}\right)^{\iota}\frac{1}{\iota} & \left(\frac{s}{\Delta}\right)^{\iota}\frac{1}{\iota} & 0 \end{bmatrix}, \qquad (\iota = 1,2,3,\cdots)$$

and

$$R^{0\iota'} = -\frac{1}{2}\begin{bmatrix} 0 & (-)^{\iota'}\,{}^{0}\bar\beta_{\iota'} & (-)^{\iota'}\,{}^{0}\bar\gamma_{\iota'} \\ {}^{0}\bar\beta_{\iota'} & 0 & {}^{0}\bar\gamma_{\iota'} \\ 0 & 0 & 0 \end{bmatrix}, \qquad (\iota' = 1,2,3,\cdots)$$

and

$$D^{\iota\iota} = -\frac{1}{2}\begin{bmatrix} \frac{1}{\iota} & (-)^{\iota}\,{}^{\iota}\bar\beta_{\iota} & {}^{\iota}\bar\gamma_{\iota} \\ (-)^{\iota}\,{}^{\iota}\bar\beta_{\iota} & \frac{1}{\iota} & {}^{\iota}\bar\gamma_{\iota} \\ {}^{\iota}\tilde\alpha_{\iota} & {}^{\iota}\tilde\alpha_{\iota} & \frac{1}{\iota} \end{bmatrix}, \qquad (i = 1,2,3,\cdots)$$

and

$$U^{\iota\iota'} = -\frac{1}{2}\begin{bmatrix} 0 & (-)^{\iota'}\,{}^{\iota}\bar\beta_{\iota'} & (-)^{\iota}(-)^{\iota'}\,{}^{\iota}\bar\gamma_{\iota'} \\ (-)^{\iota}\,{}^{\iota}\bar\beta_{\iota'} & 0 & {}^{\iota}\bar\gamma_{\iota'} \\ 0 & 0 & 0 \end{bmatrix}, \qquad (\iota,\iota' = 1,2,3,\cdots), \qquad (\iota' > \iota)$$

and

$$L^{\iota\iota'} = -\frac{1}{2}\begin{bmatrix} 0 & (-)^{\iota'}\,{}^{\iota}\bar\beta_{\iota'} & 0 \\ (-)^{\iota}\,{}^{\iota}\bar\beta_{\iota'} & 0 & 0 \\ (-)^{\iota-\iota'}\,{}^{\iota}\tilde\alpha_{\iota'} & {}^{\iota}\tilde\alpha_{\iota'} & 0 \end{bmatrix}, \qquad (\iota,\iota' = 1,2,3,\cdots), \qquad (\iota' < \iota)$$

where

$${}^{\iota'}\bar\beta_{\iota} \equiv {}^{\iota}\beta_{\iota'}\left(\frac{\delta}{2s}\right)^{\iota}\left(\frac{\delta}{2s}\right)^{\iota'}\frac{1}{\iota}$$

$${}^{\iota'}\bar\gamma_{\iota} \equiv {}^{\iota}\gamma_{\iota'}\left(\frac{s}{\Delta}\right)^{\iota}\left(\frac{\delta}{s}\right)^{\iota'}\frac{1}{\iota}$$

and

$${}^{\iota'}\bar\alpha_{\iota} \equiv {}^{\iota}\beta_{\iota'}\left(\frac{\delta}{\Delta}\right)^{\iota}\left(\frac{s}{\Delta}\right)^{\iota'}\frac{1}{\iota}$$

and

$${}^{\iota'}\tilde\alpha_{\iota} \equiv {}^{\iota'-\iota}\tilde\alpha_{\iota} = {}^{\iota}\beta_{\iota'-\iota}\left(\frac{\delta}{\Delta}\right)^{\iota}\left(\frac{s}{\Delta}\right)^{\iota'-\iota}\frac{1}{\iota}.$$

Also

$$X = \begin{bmatrix} X^0 \\ X^1 \\ X^2 \\ X^3 \\ \vdots \end{bmatrix}, \quad \text{where} \quad X^{\iota} = \begin{bmatrix} c_{\iota}' \\ c_{\iota}'' \\ s_{\iota} \end{bmatrix}$$

and

$$E = \begin{bmatrix} C^0 \\ O \\ O \\ O \\ \vdots \end{bmatrix}, \quad \text{where} \quad C^0 = \begin{bmatrix} u_0' \\ u_0'' \\ u_0 \end{bmatrix} V_0, \quad \text{and} \quad O = [0].$$

If the ratio of the wire diameter relative to the wire separation is small and if the ratio of the wire separation relative to the shield diameter is small, then the off-diagonal terms in the matrix $\boldsymbol{M}$, which contain products of various powers of these ratios, decay away from the main diagonal of the matrix. Also for the same reasons, the terms on the main diagonal of the matrix $\boldsymbol{M}$ decay (from upper left to lower right) along the main diagonal of the matrix. Therefore, only those terms in the first few partitions (beginning at upper left) are significant in the matrix $\boldsymbol{M}$. Accurate approximations to the inverse of the infinite matrix $\boldsymbol{M}$ are obtained by truncating the infinite matrix to a square matrix which contains only the significant terms in the infinite matrix.

Therefore, approximate expressions for the Fourier coefficients of the various surface-charge densities are determined for any desired degree of accuracy.

V. CAPACITANCE DEFINITIONS

The corresponding line-charge densities $\lambda_{\left\{ {1 \atop 2} \right.}$ (per unit length in the axial direction) due to the applied voltages $V_{\left\{ {1 \atop 2} \right.}$ on wires 1 and 2 are determined by

$$\lambda_{\left\{ {1 \atop 2} \right.} = \oint_C dl \, \sigma^{\left\{ {\prime \atop \prime\prime} \right.} = \rho_0 \int_0^{2\pi} d\varphi \sigma^{\left\{ {\prime \atop \prime\prime} \right.}$$

where $\sigma','' $ are the resulting surface-charge densities on wires 1 and 2. In terms of the Fourier coefficients of the surface-charge densities

$$\lambda_{\left\{ {1 \atop 2} \right.} = \lambda_0 c_0^{\left\{ {\prime \atop \prime\prime} \right.}.$$

Therefore, the various line capacitances can be expressed in terms of the Fourier coefficients as

$$C_{11} \equiv \left. \frac{\lambda_1}{V_1} \right|_{\substack{V_1 = V_0 \\ V_2 = 0}} = \lambda_0 \left. \frac{c_0'}{V_1} \right|_{\substack{V_1 = V_0 \\ V_2 = 0}} = -2\pi\epsilon m_{11}^{-1}$$

$$C_{22} \equiv \left. \frac{\lambda^2}{V_2} \right|_{\substack{V_1 = 0 \\ V_2 = V_0}} = \lambda_0 \left. \frac{c_0''}{V_2} \right|_{\substack{V_1 = 0 \\ V_2 = V_0}} = -2\pi\epsilon m_{22}^{-1}$$

and

$$C_{12} \equiv \left. \frac{\lambda_1}{V_2} \right|_{\substack{V_1 = 0 \\ V_2 = V_0}} = \lambda_0 \left. \frac{c_0'}{V_2} \right|_{\substack{V_1 = 0 \\ V_2 = V_0}} = -2\pi\epsilon m_{12}^{-1}$$

$$C_{21} \equiv \left. \frac{\lambda_2}{V_1} \right|_{\substack{V_1 = V_0 \\ V_2 = 0}} = \lambda_0 \left. \frac{c_0''}{V_1} \right|_{\substack{V_1 = V_0 \\ V_2 = 0}} = -2\pi\epsilon m_{21}^{-1}$$

where $m_{11}^{-1}, m_{12}^{-1}, m_{21}^{-1}, m_{22}^{-1}$ are the terms of the first 2×2 partition of the inverse $\boldsymbol{M}^{-1}$ of the matrix $\boldsymbol{M}$.

By reciprocity, $C_{12} = C_{21}$; and, by symmetry, $C_{11} = C_{22}$.

The line capacitance C_d directly between the wires is defined by

$$C_d = -C_{12} = -C_{21}$$

and the line capacitance C_g to ground is defined by

$$C_g = C_{11} + C_{12} = C_{22} + C_{21}.$$

Also, the mutual line capacitance C_m is defined by

$$C_m = C_d + \tfrac{1}{2} C_g = \frac{C_{11} - C_{12}}{2} = \frac{C_{22} - C_{21}}{2}.$$

VI. RESULTS

The foregoing theoretical results are now applied to determine the Fourier coefficients of the surface-charge densities on the inner conductors and the shield and the various capacitances associated with the cable structure for a realistic cable geometry with various impressed voltage excitations. In particular, both balanced and longitudinal voltage excitations on a typical standard production cable using 22-gauge wires are considered. The pertinent geometrical parameters for this cable are shown to scale in Fig. 2.

This cable is an equivalent shielded-pair model for one pair of a 50-pair PIC cable manufactured by Western Electric. In the actual cable, a thin annular layer of insulation surrounds each wire and the space between the insulation and shield is filled with air and other pairs of the 22-gauge PIC wires. The insulations and air spaces have different dielectric constants; however, for simplicity, this inhomogeneous dielectric between the wires is replaced with an equivalent homogeneous dielectric.

The effective relative permittivity of an equivalent uniform dielectric surrounding the 22-gauge wires, as determined indirectly from a method based on previous capacitance measurements, is

$$\epsilon_r = 2.026.$$

Table I contains the values of the capacitance to ground, the direct capacitance, and the mutual capacitance, in addition to the four elements of the capacitance matrix, for the cable geometry described previously.

Tables II and III contain the first ten Fourier cosine coefficients of the surface-charge densities on the inner conductors and the shield for the cable geometry and the voltage excitations described previously.

Figs. 3 and 4 contain plots of the surface-charge densities on the inner conductors and the shield, as constructed from their Fourier coefficients, for the cable geometry and the voltage excitations already described.

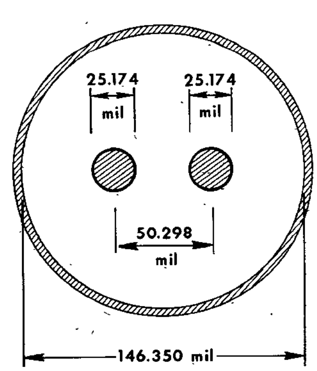

Fig. 2. Cross section of a cable using 22-gauge wires.

TABLE I
CAPACITANCE MATRIX FOR 22-GAUGE WIRE (F/M)

c_{11}	0.775(-10)
c_{12}	-0.226(-10)
c_{21}	-0.226(-10)
c_{22}	0.775(-10)
c_d	0.226(-10)
c_g	0.549(-10)
c_m	0.500(-10)

TABLE II
FOURIER COSINE COEFFICIENTS FOR 22-GAUGE WIRE AND BALANCED EXCITATION (F/M)

INDEX	WIRE 1	WIRE 2	SHIELD
0	-0.10009340(-09)	0.10009340(-09)	-0.16263032(-18)
1	-0.30043817(-10)	-0.30043817(-10)	-0.12726578(-09)
2	-0.13937212(-10)	0.13937212(-10)	-0.27105054(-19)
3	-0.35085716(-11)	-0.35085716(-11)	-0.13405334(-10)
4	-0.94407508(-12)	0.94407508(-12)	-0.67762635(-20)
5	-0.24811336(-12)	-0.24811335(-12)	-0.14941702(-11)
6	-0.65645312(-13)	0.65645312(-13)	-0.84703294(-21)
7	-0.17378892(-13)	-0.17378892(-13)	-0.17531177(-12)
8	-0.46111371(-14)	0.46111371(-14)	-0.15881867(-21)
9	-0.12255219(-14)	-0.12255219(-14)	-0.21428396(-13)
BALANCED EXCITATION			

TABLE III
FOURIER COSINE COEFFICIENTS FOR 22-GAUGE WIRE AND LONGITUDINAL EXCITATION (F/M)

INDEX	WIRE 1	WIRE 2	SHIELD
0	0.54894168(-10)	0.54894168(-10)	-0.10978833(-09)
1	-0.27595980(-10)	0.27595980(-10)	-0.21684043(-18)
2	-0.49470636(-11)	-0.49470636(-11)	-0.32168844(-10)
3	-0.13874647(-11)	0.13874647(-11)	-0.67762635(-20)
4	-0.30369678(-12)	-0.30369678(-12)	-0.44165031(-11)
5	-0.68629928(-13)	0.68629928(-13)	-0.25410988(-20)
6	-0.14829190(-13)	-0.14829190(-13)	-0.58428940(-12)
7	-0.31145136(-14)	0.31145136(-14)	-0.21175823(-21)
8	-0.62017627(-15)	-0.62017626(-15)	-0.75566852(-13)
9	-0.11322906(-15)	0.11322906(-15)	-0.26469779(-22)
LONGITUDINAL EXCITATION			

VII. DISCUSSION

The results of this theoretical analysis were compared to the results obtained from a purely numerical analysis of this problem using a field-mapping program. In particular, the first ten Fourier coefficients of the surface-charge densities on the wires and the shield were compared. The zeroth-order Fourier coefficients on the shield agreed to three significant digits, which represents a discrepancy of less than 0.5 percent; the zeroth-order coefficients on the wires agreed to two significant digits, which represents a discrepancy of less than 1.0 percent. For the higher order coefficients the errors increased somewhat; however, the errors in the higher order coefficients are less significant, since the coefficients decay monotonically with increasing indices, roughly, one order of magnitude per index, and are negligible in comparison to the zeroth-order coefficients.

After an examination of Table I, it is found that the capacitance to ground, the direct capacitance, and the mutual capacitance agree with previously measured data. For example, the mutual capacitance C_m of 0.0500 pF/m is approximately 1.0 percent greater than the measured value of 0.0495 pF/m.

After an examination of Tables II and III, it is found that the Fourier coefficients of the surface-charge densities on the inner conductors are greater for the case of balanced excitation than for the case of longitudinal excitation, since the potential difference between the wires is zero for longitudinal excitation. For these special cases of excitation, the resulting surface-charge densities are even functions of the azimuthal angles describing the circumferences of each conductor; and, therefore, the Fourier

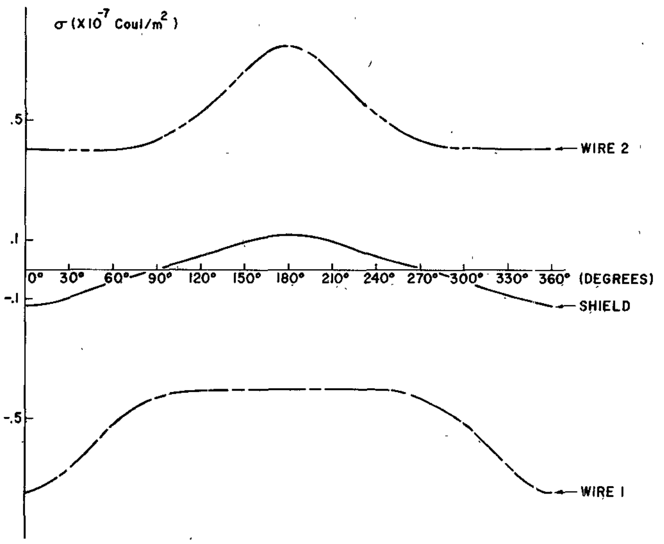

Fig. 3. Surface-charge density versus azimuthal angle for 22-gauge wire and balanced excitation.

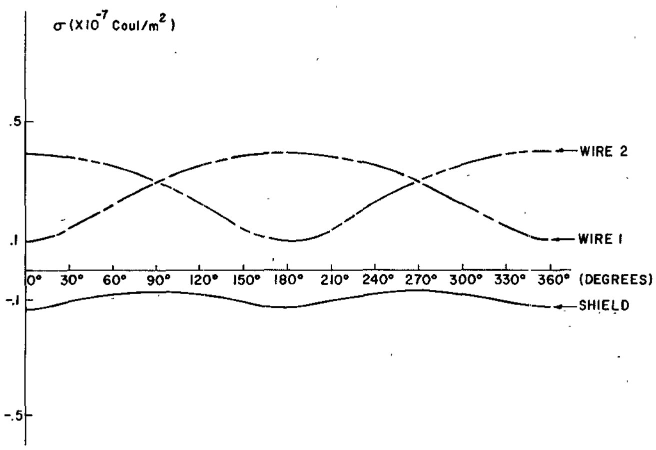

Fig. 4. Surface-charge density versus azimuthal angle for 22-gauge wire and longitudinal excitation.

sine coefficients are zero. Notice that on the shield the even harmonics are negligible (zero) for the case of balanced excitation and the odd harmonics are negligible (zero) for the case of longitudinal excitation; this condition of alternating zero harmonics is imposed by the horizontal and vertical symmetry (or antisymmetry) of the cable and the excitation.

VIII. CONCLUSIONS

In this paper, the capacitance matrix of a straight pair of wires in a shield was determined theoretically. The Fourier coefficients of the surface-charge densities on the inner conductors and the shield and the various capacitances associated with the cable structure were then determined. The cable structure was constrained to be symmetric about the axis of the shield; however, the voltage excitation was completely arbitrary. Therefore, both balanced and longitudinal excitations were considered. The theoretical results were evaluated numerically for the case of a typical standard production cable using 22-gauge wires.

The Fourier coefficients of the surface-charge densities are required in a recently developed method for determining the propagation parameters and the associated propagation modes of the cable, for either the straight or the twisted case, and for either a balanced or an unbalanced geometry.

The results of this paper for the case of a straight pair of wires in a shield will be compared to the results obtained in a subsequent paper for the case of a twisted pair of wires in a shield. Also, the model will be extended to include certain types of asymmetries in the cable geometry.

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